Period 4, Oct 14, 2024

5. Find $\frac{d^2y}{dx^2}$ in terms of x and y if $xy + y^2 = 4$.

$$xy + y^2 = 4$$

$$1 \cdot y + x \cdot \frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$x\frac{dy}{dx} + 2y\frac{dy}{dx} = -y$$

$$\frac{dy}{dx}(x+2y) = -y$$

$$\boxed{\frac{dy}{dx} = \frac{-y}{(x+2y)}}$$

$$\frac{dy}{dx} = \frac{-y}{(x+2y)}$$

$$\frac{d^2y}{dx^2} = \frac{-1\frac{dy}{dx}(x+2y) - (-y)(1+2\frac{dy}{dx})}{(x+2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{-\frac{dy}{dx}(x+2y) + y(1+2\frac{dy}{dx})}{(x+2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{-\left(\frac{-y}{\cancel{(x+2y)}}\right)\cancel{(x+2y)} + y + 2y \cdot \frac{-y}{(x+2y)}}{(x+2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{(x+2y)}{(x+2y)}2y - \frac{2y^2}{(x+2y)}}{(x+2y)^2} = \frac{d^2y}{dx^2} = \frac{\frac{2xy+4y^2-2y^2}{(x+2y)}}{\frac{(x+2y)^2}{1}}$$

$$\frac{2xy+2y^2}{(x+2y)} \cdot \frac{1}{(x+2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{2y(x+y)}{(x+2y)^3}$$

d) $\sin^2 y + \cos^2 y = y + 2$

$$2\sin y(\cos y)\cdot\frac{dy}{dx} + 2\cos y(-\sin y)\cdot\frac{dy}{dx} = \frac{dy}{dx} + 0$$

$$1 = y + 2$$

$$0 = \frac{dy}{dx} + 0$$

b. $x^3 + 5x^2y + 2y^2 = 4y + 11$ at $(1,2)$.

$$3x^2 + 10x\cdot y + 5x^2\frac{dy}{dx} + 4y\frac{dy}{dx} = 4\frac{dy}{dx}$$

$$-5x^2\frac{dy}{dx} \quad -4y\frac{dy}{dx} \quad -5x^2\frac{dy}{dx} \quad -4y\frac{dy}{dx}$$

$$3x^2 + 10xy = 4\frac{dy}{dx} - 5x^2\frac{dy}{dx} - 4y\frac{dy}{dx} \Rightarrow 3(1)^2 + 10(1)(2) = 4\frac{dy}{dx} - 5(1)^2\frac{dy}{dx} - 4(2)\frac{dy}{dx}$$

$$3 + 20 = (4 - 5 - 8)\frac{dy}{dx}$$

$$23 = -9\frac{dy}{dx}$$

$$\frac{3x^2 + 10xy}{4 - 5x^2 - 4y} = \frac{dy}{dx}\frac{\cancel{(4 - 5x^2 - 4y)}}{\cancel{4 - 5x^2 - 4y}}$$

$$\frac{3x^2 + 10xy}{4 - 5x^2 - 4y} = \frac{dy}{dx} = \frac{3(1)^2 + 10(1)(2)}{4 - 5(1)^2 - 4(2)} = \frac{3 + 20}{4 - 5 - 8} = \frac{23}{-9} = -\frac{23}{9}$$

3. Find the slope of the curve at the given point.

$$y = \sqrt{5x^3 - 6x^2} = (5x^3 - 6x^2)^{\frac{1}{2}}$$

$$u = 5x^3 - 6x^2 \Rightarrow y = u^{\frac{1}{2}}$$

$$\frac{du}{dx} = 15x^2 - 12x \qquad \frac{dy}{du} = \frac{1}{2}u^{-\frac{1}{2}}$$

$$\frac{du}{dx}\frac{dy}{du} = (15x^2 - 12x)\left(\frac{1}{2\sqrt{5x^3 - 6x^2}}\right)$$

a. $\sqrt{xy} = 1$ at $\left(2, \frac{1}{2}\right)$.

$$(xy)^{\frac{1}{2}} = 1$$

$$\frac{1}{2}(xy)^{\frac{1}{2}-1}\left(1\cdot y + x\cdot\frac{dy}{dx}\right) = 0$$

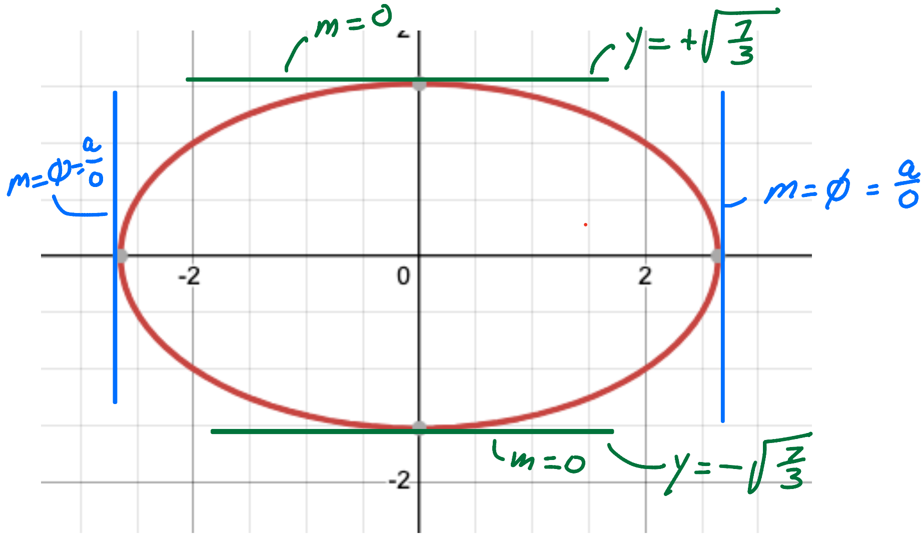
$$\frac{1}{2\sqrt{xy}}\cdot\left(y + x\frac{dy}{dx}\right) = 0$$

$$\frac{y}{2\sqrt{xy}} + \frac{x}{2\sqrt{xy}}\frac{dy}{dx} = 0 \Rightarrow$$

$$\cancel{\frac{2\sqrt{xy}}{x}}\cdot\frac{\cancel{x}}{\cancel{2\sqrt{xy}}}\frac{dy}{dx} = \frac{-y}{\cancel{2\sqrt{xy}}}\cdot\cancel{\frac{2\sqrt{xy}}{x}}$$

$$\frac{dy}{dx} = \frac{-y}{x}$$

$$(x^{\frac{1}{2}}\cdot y^{\frac{1}{2}}) = 1 \Rightarrow \frac{1}{2}\cdot x^{-\frac{1}{2}}\cdot y^{\frac{1}{2}} + x^{\frac{1}{2}}\cdot\frac{1}{2}y^{-\frac{1}{2}}\frac{dy}{dx} = 0 \Rightarrow \cancel{\frac{-2\sqrt{y}}{\sqrt{x}}}\frac{\sqrt{y}}{2\sqrt{x}} = -\frac{\cancel{\sqrt{x}}}{\cancel{2\sqrt{y}}}\frac{dy}{dx}\cdot\cancel{\frac{-2\sqrt{y}}{\sqrt{x}}}$$



6. Find the points where the graph of $x^2 + 3y^2 = 7$ has a horizontal tangent line.

$$2x+6y\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=\frac{-2x}{6y}=\frac{-x}{3y}$$

$m=0=\frac{0}{a}$

X must Equal 0

$x^2+3y^2=7$

$0^2+3y^2=7 \Rightarrow 3y^2=7 \Rightarrow y^2=\frac{7}{3} \Rightarrow y=\pm\frac{\sqrt{7}}{\sqrt{3}}$

7. Find the points where the graph of $x^2 + 3y^2 = 7$ has a vertical tangent line.

$y=0$

$$y = \cos(5x^3 - 6x^2) \Rightarrow y = \cos u$$

$u = 5x^3 - 6x^2$ $\qquad \frac{dy}{du} = -\sin u$

$\frac{du}{dx} = 15x^2 - 12x$

$$\frac{du}{dx} \cdot \frac{dy}{du} = \frac{dy}{dx} = (5x^2 - 12)(-\sin(5x^3 - 6x^2))$$

$3 = \cos xy$

$0 = (-\sin xy)(1 \cdot y + x \cdot \frac{dy}{dx})$

$3 = \cos xy \Rightarrow 3 = \cos u$

$u = xy$ $\qquad 0 = (-\sin u)\frac{du}{dx} \Rightarrow 0 = (-\sin xy)(y + x\frac{dy}{dx})$

$\frac{du}{dx} = 1 \cdot y + x \cdot \frac{dy}{dx}$

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1) $F(x) = 5x^4 - 3x^8 + 6x^2 - e^x$

$F'(x) = 20x^3 - 24x^7 + 2x - e^x$

1) $F(x) = 7x^4 - 3x^5 + 6x^3 - 3e^x$

$F'(x) = 28x^3 - 15x^4 + 18x^2 - 3e^x$

2) $F(x) = \dfrac{5x^3 + 7x^{\frac{1}{2}} - 4x}{\sqrt[4]{x}} = \dfrac{5x^3}{x^{\frac{1}{4}}} + \dfrac{7x^{\frac{1}{2}}}{x^{\frac{1}{4}}} - \dfrac{4x}{x^{\frac{1}{4}}} = 5x^{2\frac{3}{4}} + 7x^{\frac{1}{4}} - 4x^{\frac{3}{4}} \Rightarrow F'(x) = \dfrac{55}{4}x^{1\frac{3}{4}} + \dfrac{7}{4}x^{-\frac{3}{4}} - 3x^{-\frac{1}{4}}$

$x^{\frac{1}{4}}$

$$F'(x) = \frac{\left(15x^2 + \frac{7}{2\sqrt{x}} - 4\right)\left(\sqrt[4]{x}\right) - \left(5x^3 + 7x^{\frac{1}{2}} - 4x\right)\left(\frac{1}{4\sqrt[4]{x^3}}\right)}{\left(\sqrt[4]{x}\right)^2}$$

$$\frac{15x^{2\frac{1}{4}} + \frac{7}{2x^{\frac{1}{4}}} - 4x^{\frac{1}{4}}}{\sqrt{x}} - \frac{\frac{5x^3 + 7x^2 - 4x}{4\sqrt[4]{x^3}}}{\frac{\sqrt{x}}{1}}$$

$$\frac{15x^{2\frac{1}{4} - \frac{1}{2}} + \frac{7}{2}x^{-\frac{1}{4} - \frac{1}{2}} - 4x^{\frac{1}{4} - \frac{1}{2}} - \frac{(5x^3 + 7x^2 - 4x)}{4\sqrt[4]{x^3} \cdot \sqrt{x}}}{}$$

$F(x) = \dfrac{4x^3 + 6x^{\frac{1}{2}} - x}{\sqrt[3]{x}}$

$$\frac{\left(12x^2 + \frac{3}{\sqrt{x}} - 1\right)\left(\sqrt[3]{x}\right) - \left(4x^3 + 6x^{\frac{1}{2}} - x\right)\left(\frac{1}{3}x^{-4/3}\right)}{\left(\sqrt[3]{x}\right)^2}$$

$$\frac{\frac{12x^{2\frac{1}{3}} + 3 - \sqrt{x}}{\sqrt[3]{x^2} \cdot \sqrt{x}} - \frac{4x^3 + 6\sqrt{x} - x}{3x^{4/3}}}{}$$

3)

$$F(x) = (12x^{-2}+6x^{3})(8x^{5}-7x^{2}+3)$$

$$(-24x^{-3}+18x^{2})(8x^{5}-7x^{2}+3)+(12x^{-2}+6x^{3})(8x^{5}-7x^{2}+3)$$

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3) $F(x) = (8x^{-3}+2x^{5})(3x^{6}-7x^{2}+5)$

$$F'(x) = (-24x^{-4}+10x^{4})(3x^{6}-7x^{2}+5)+(8x^{3}+2x^{5})(18x^{5}-14x)$$

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4)

$$F(x) = \frac{(9x^{2}+4)(7x^{5}-3x^{-1})}{(2x^{3}+7)} \Rightarrow F(x) = \frac{63x^{7}-27x+28x^{5}-12x^{-1}}{(2x^{3}+7)^{2}}$$

$$F'(x) = \frac{\left[18x(7x^{5}-3x^{-1})+(9x^{2}+4)(35x^{4}+3x^{-2})\right](2x^{3}+7)-(9x^{2}+4)(7x^{5}-3x^{-1})(6x^{2})}{(2x^{3}+7)^{2}}$$

$$F'(x) = \frac{(441x^{6}-27+140x^{4}+12x^{-2})(2x^{3}+7)-(63x^{7}-27x+28x^{5}-12x^{-1})(6x^{2})}{(2x^{3}+7)^{2}}$$

4) $F(x)=\dfrac{(4x^2+2)(7x^4-3x^{-2})}{(2x^3+1)}=\dfrac{28x^6-12+14x^4-6x^{-2}}{(2x^3+1)}$

$$F'(x)=\frac{\left[(8x)(7x^4-3x^{-2})+(4x^2+2)(28x^3+6x^{-3})\right](2x^3+1)-(4x^2+2)(7x^4-3x^{-2})(6x^2)}{(2x^3+1)^2}$$

$$F'(x)=\frac{(168x^5+56x^3+12x^{-3})(2x^3+1)-(28x^6-12+14x^4-6x^{-2})(6x^2)}{(2x^3+1)^2}$$